# dfidx

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In some situations, series from a data frame have a natural two-dimensional (tabular) representation because each observation can be uniquely characterized by a combination of two indexes. Two major cases of these situations in applied econometrics are:

- panel data, where the same individuals are observed for several time periods,
- random utility models, where each observation describes the features of an alternative among a set of alternatives for a given choice situation.

The idea of **dfidx** is to keep in the same object the data and the information about its structure. A **dfidx** object is a data frame with an **idx** column, which is a data frame that contains the series that define the indexes.

From version 0.1-2, **dfidx** doesn't depend anymore on some of **tidyverse** packages. If you want to use **dfidx** along with **tidyverse** in order to use tibbles instead of ordinary data frames and **dplyr**'s verbs, you should use the new **tidydfidx** package instead of **dfidx**.

#### 1 Basic use of the dfidx function

The dfidx package is loaded using:

```
library(dfidx)
```

To illustrate the features of **dfidx**, we'll use the munnell data set (Munnell 1990) that is used in Baltagi (2013)'s famous book and is part of the **plm** package as **Produc**. It contains several economic series for American states from 1970 to 1986. We've added to the initial data set a **president** series which indicates the name of the American president in power for the given year.

```
head(munnell, 3)
```

```
state year
                           region president publiccap highway
1 Alabama 1970 East-South Central
                                       Nixon
                                              15032.67 7325.80 1655.68
2 Alabama 1971 East-South Central
                                              15501.94 7525.94 1721.02
                                       Nixon
3 Alabama 1972 East-South Central
                                              15972.41 7765.42 1764.75
                                       Nixon
  utilities privatecap
                         gsp labor unemp
              35793.80 28418 1010.5
1
    6051.20
                                       4.7
2
    6254.98
              37299.91 29375 1021.9
                                       5.2
3
    6442.23
              38670.30 31303 1072.3
                                       4.7
```

The two indexes are state and year and both are nested in another variable: state in region and year in president. A dfidx object is created with the dfidx function: the first argument should be a data frame (or a tibble) and the second argument idx is used to indicate the indexes. As, in the munnell data set, the first two columns contain the two indexes, the idx argument is not mandatory and a dfidx can be obtained from the munnell tibble simply by using:

```
dfidx(munnell) > print(n = 3)
-- -- -- -- --
first 3 observations out of 816
              region president publiccap highway
                                                     water utilities
                          Nixon 15032.67 7325.80 1655.68
1 East-South Central
                                                             6051.20
2 East-South Central
                                 15501.94 7525.94 1721.02
                                                             6254.98
                          Nixon
3 East-South Central
                          Nixon
                                 15972.41 7765.42 1764.75
                                                             6442.23
               gsp labor unemp
 privatecap
1
    35793.80 28418 1010.5
                             4.7 Alab:1970
    37299.91 29375 1021.9
                             5.2 Alab:1971
2
    38670.30 31303 1072.3
                             4.7 Alab:1972
~~~ indexes ~~~~
    state year
1 Alabama 1970
2 Alabama 1971
3 Alabama 1972
indexes: 1, 2
```

The resulting object is of class dfidx and is a tibble with an idx column, which is a tibble containing the two indexes. Note that the two indexes are now longer standalone series in the resulting tibble, because the default value of the drop.index argument is TRUE. The header of the tibble indicates the names and the cardinal of the two indexes. It also indicated whether

the data set is balanced ie, in this panel data context, whether all the states are observed for the same set of years (which is the case for the munnell data set). The idx column can be retrieved using the idx function:

```
munnell |> dfidx() |> idx() |> print(n = 3)

~~~ indexes ~~~~
    state year
1 Alabama 1970
2 Alabama 1971
3 Alabama 1972
indexes: 1, 2
```

If the first two columns don't contain the indexes, the idx argument should be set. If the observations are ordered first by the first index and then by the second one and if the data set is *balanced*, idx can be an integer, the number of distinct values of the first index:

```
munnell > dfidx(48) > print(n = 3)
first 3 observations out of 816
~~~~~~
                           region president publiccap highway
    state year
1 Alabama 1970 East-South Central
                                      Nixon 15032.67 7325.80 1655.68
                                      Nixon 15501.94 7525.94 1721.02
2 Alabama 1971 East-South Central
3 Alabama 1972 East-South Central
                                      Nixon 15972.41 7765.42 1764.75
                         gsp labor unemp idx
 utilities privatecap
    6051.20
              35793.80 28418 1010.5
                                      4.7 1:1
1
2
    6254.98
              37299.91 29375 1021.9
                                      5.2 1:2
3
    6442.23
              38670.30 31303 1072.3
                                      4.7 1:3
~~~ indexes ~~~~
  id1 id2
1
    1
        1
    1
        2
2
    1
        3
indexes: 1, 2
```

Then the two indexes are created with the default names id1 and id2. More relevant names can be indicated using the idnames argument and the values of the second index can be indicated, using the levels argument.

```
munnell |> dfidx(48, idnames = c("state", "year"),
                   levels = 1970:1986) |> print(n = 3)
~~~~~~
first 3 observations out of 816
             region president publiccap highway water utilities
1 East-South Central
                        Nixon 15032.67 7325.80 1655.68
                                                          6051.20
2 East-South Central
                        Nixon 15501.94 7525.94 1721.02
                                                          6254.98
                        Nixon 15972.41 7765.42 1764.75
3 East-South Central
                                                          6442.23
 privatecap
              gsp labor unemp
   35793.80 28418 1010.5
                           4.7 1:1970
   37299.91 29375 1021.9
2
                           5.2 1:1971
  38670.30 31303 1072.3 4.7 1:1972
~~~ indexes ~~~~
 state year
     1 1970
     1 1971
     1 1972
indexes: 1, 2
```

The idx argument can also be a character of length one or two. In the first case, only the first index is indicated:

```
munnell |> dfidx("state", idnames = c(NA, "date"),
                  levels = 1970:1986) |> print(n = 3)
first 3 observations out of 816
                  region president publiccap highway
 year
1 1970 East-South Central
                             Nixon 15032.67 7325.80 1655.68
2 1971 East-South Central
                                   15501.94 7525.94 1721.02
                             Nixon
3 1972 East-South Central
                             Nixon 15972.41 7765.42 1764.75
 utilities privatecap gsp labor unemp
                                               idx
   6051.20
             35793.80 28418 1010.5
                                    4.7 Alab:1970
2
   6254.98
             37299.91 29375 1021.9 5.2 Alab:1971
  6442.23
             38670.30 31303 1072.3 4.7 Alab:1972
```

```
state date
1 Alabama 1970
2 Alabama 1971
3 Alabama 1972
indexes: 1, 2
```

Note that we've only provided a name for the second index, the NA in the first position of the idnames argument meaning that we want to keep the original name for the first index. Finally, if the idx argument is a character of length 2, it should contain the name of the two indexes.

```
munnell |> dfidx(c("state", "year")) |> print(n = 3)
first 3 observations out of 816
~~~~~~
              region president publiccap highway
                                                    water utilities
1 East-South Central
                         Nixon
                                15032.67 7325.80 1655.68
                                                            6051.20
2 East-South Central
                         Nixon
                                15501.94 7525.94 1721.02
                                                            6254.98
3 East-South Central
                                15972.41 7765.42 1764.75
                                                            6442.23
                         Nixon
               gsp labor unemp
                                       idx
 privatecap
                            4.7 Alab:1970
1
    35793.80 28418 1010.5
2
                            5.2 Alab:1971
    37299.91 29375 1021.9
3
    38670.30 31303 1072.3
                            4.7 Alab:1972
~~~ indexes ~~~~
    state year
1 Alabama 1970
2 Alabama 1971
3 Alabama 1972
indexes: 1, 2
```

## 2 More advanced use of dfidx

## 2.1 Nesting structure

One or both of the indexes may be nested in another series. In this case, the idx argument is still a character of length two, but the nesting series is indicated as the name of the corresponding index:

```
mn <- munnell |> dfidx(c(region = "state", "year"))
  mn <- munnell |> dfidx(c(region = "state", president = "year"))
  mn \mid > print(n = 3)
first 3 observations out of 816
 publiccap highway
                       water utilities privatecap
                                                     gsp labor unemp
1 62200.83 26835.52 7669.60 27695.71
                                         146285.6 168627 4655.5
                                                                  6.2
                                         150855.5 173767 4788.8
2 63096.48 27300.22 8004.98 27791.28
                                                                  6.0
3 63642.81 27247.22 8491.35 27904.24
                                        156752.0 173817 4880.0
                                                                  5.5
1 Illi:1977
2 Illi:1978
3 Illi:1979
~~~ indexes ~~~~
     state
                       region year president
1 Illinois East-North Central 1977
                                      Carter
2 Illinois East-North Central 1978
                                      Carter
3 Illinois East-North Central 1979
                                      Carter
indexes: 1, 1, 2, 2
```

The idx column is now a tibble containing the two indexes and the nesting variables.

### 2.2 Customized the name and the position of the idx column

By default, the column that contains the indexes is called idx and is the first column of the returned data frame. The position and the name of this column can be set using the position and name arguments:

```
dfidx(munnell, idx = c(region = "state", president = "year"),
              name = "index", position = 4) |> print(n = 3)
~ ~ ~ ~ ~ ~ ~
first 3 observations out of 816
 publiccap highway
                                 index utilities privatecap
                       water
1 62200.83 26835.52 7669.60 Illi:1977
                                         27695.71
                                                    146285.6 168627
2 63096.48 27300.22 8004.98 Illi:1978
                                       27791.28
                                                    150855.5 173767
3 63642.81 27247.22 8491.35 Illi:1979 27904.24
                                                    156752.0 173817
  labor unemp
1 4655.5
           6.2
2 4788.8
           6.0
3 4880.0
           5.5
~~~ indexes ~~~~
     state
                       region year president
1 Illinois East-North Central 1977
                                      Carter
2 Illinois East-North Central 1978
                                      Carter
3 Illinois East-North Central 1979
                                      Carter
indexes: 1, 1, 2, 2
```

#### 2.3 Data frames in wide format

dfidx can deal with data frames in wide format, i.e., for which each series for a given value of the second index is a column of the data frame. This is the case of the munnell\_wide tibble that contains two series of the original data set (gsp and unemp).

```
head(munnell_wide, 3)
```

```
region gsp_1970 gsp_1971 gsp_1972 gsp_1973
     state
1 Alabama East-South Central
                                  28418
                                            29375
                                                     31303
                                                               33430
2 Arizona
                      Montain
                                  19288
                                            21040
                                                     23289
                                                               25244
3 Arkansas West-South Central
                                  15392
                                            16177
                                                     17702
                                                               18825
  gsp_1974 gsp_1975 gsp_1976 gsp_1977 gsp_1978 gsp_1979 gsp_1980
1
     33749
              33604
                        35764
                                 37463
                                           39964
                                                    40979
                                                              40380
2
     25698
              24915
                                 28110
                                           31062
                                                    33943
                                                             34708
                        26041
3
     19287
              19024
                        20277
                                 21410
                                           23063
                                                    23501
                                                             23210
  gsp_1981 gsp_1982 gsp_1983 gsp_1984 gsp_1985 gsp_1986 unemp_1970
     41105
              40328
                        42245
                                 45118
                                           46849
                                                    48409
                                                                  4.7
```

```
2
     35244
               33603
                         35963
                                   40010
                                             43350
                                                       46058
                                                                      4.4
3
     24134
               23462
                         24415
                                   26512
                                             27159
                                                       28168
                                                                      5.0
  unemp_1971 unemp_1972 unemp_1973 unemp_1974 unemp_1975 unemp_1976
                      4.7
                                                           7.7
1
         5.2
                                  3.9
                                              5.5
                                                                       6.8
2
         4.7
                      4.2
                                  4.1
                                              5.6
                                                          12.0
                                                                       9.8
3
         5.4
                      4.6
                                                                       7.1
                                  4.1
                                              4.8
                                                           9.5
  unemp_1977 unemp_1978 unemp_1979 unemp_1980 unemp_1981 unemp_1982
1
         7.4
                      6.3
                                  7.1
                                              8.8
                                                          11.0
                                                                      14.0
2
         8.2
                      6.1
                                  5.1
                                                           6.1
                                                                       9.9
                                              6.7
3
         6.6
                      6.4
                                  6.2
                                              7.6
                                                           9.1
                                                                       9.8
  unemp_1983 unemp_1984 unemp_1985 unemp_1986
1
         14.0
                     11.0
                                  8.9
                                              9.8
2
                                              6.9
         9.1
                      5.0
                                  6.5
3
        10.0
                      8.9
                                  8.7
                                              8.7
```

Each line is now an American state and, apart the indexes, there are now 34 series with names obtained by the concatenation of the name of the series and the year (for example gsp\_1988). In this case a supplementary argument called varying should be provided. It is a vector of integers indicating the position of the columns that should be merged in the resulting long formatted data frame. The stats::reshape function is then used and the sep argument can be also provided to indicate the separating character in the names of the series (the default value being ".").

```
munnell_wide |> dfidx(varying = 3:36, sep = "_") |> print(n = 3)
~~~~~~
first 3 observations out of 816
~~~~~~
    state
                      region
                                gsp unemp
                                             idx
1 Alabama East-South Central 28418
                                      4.7 1:1970
2 Alabama East-South Central 29375
                                      5.2 1:1971
3 Alabama East-South Central 31303
                                      4.7 1:1972
~~~ indexes ~~~~
  id1 id2
    1 1970
1
2
    1 1971
3
    1 1972
indexes: 1, 2
```

Better results can be obtained using the idx and idnames previously described:

```
munnell_wide |> dfidx(idx = c(region = "state"), varying = 3:36,
                        sep = "_", idnames = c(NA, "year")) |>
      print(n = 3)
first 3 observations out of 816
                     idx
     gsp unemp
1 145792
           4.1 Illi:1970
2 148503
           5.1 Illi:1971
3 154413
           5.1 Illi:1972
~~~ indexes ~~~~
     state
                       region year
1 Illinois East-North Central 1970
2 Illinois East-North Central 1971
3 Illinois East-North Central 1972
indexes: 1, 1, 2
```

## 3 Getting the indexes or their names

The name (and the position) of the idx column can be obtained as a named integer (the integer being the position of the column and the name its name) using the idx\_name function:

```
idx_name(mn)
## idx
## 9
```

To get the name of one of the indexes, the second argument, n, is set either to 1 or 2 to get the first or the second index, ignoring the nesting variables:

```
idx_name(mn, 2)
## [1] "year"
idx_name(idx(mn), 2)
## [1] "year"
```

Not that idx\_name can be in this case applied to a dfidx or to a idx object. To get a nesting variable, the third argument, called m, is set to 2:

```
idx_name(mn, 1, 1)
## [1] "state"
idx_name(mn, 1, 2)
## [1] "region"
```

To extract one or all the indexes, the idx function is used. This function has already been encountered when one wants to extract the idx column of a dfidx object. The same n and m arguments as for the idx\_name function can be used in order to extract a specific series. For example, to extract the region index, which nests the state index:

```
id_index1 <- idx(mn, n = 1, m = 2)
id_index2 <- idx(idx(mn), n = 1, m = 2)
head(id_index1)
## [1] "East-North Central" "East-North Central" "East-North Central"
## [4] "East-North Central" "East-North Central" "East-North Central"
identical(id_index1, id_index2)
## [1] TRUE</pre>
```

## 4 Data frames subsetting

Subsets of data frames are obtained using the [ and the [[ operators. The former returns most of the time a data frame as the second one always returns a series.

#### 4.1 Commands that return a data frame

Consider first the use of [. If one argument is provided, it indicates the columns that should be selected. The result is always a data frame, even if a single column is selected. If two arguments are provided, the first one indicates the subset of lines and the second one the subset of columns that should be returned. If only one column is selected, the result depends on the value of the drop argument. If TRUE, a series is returned and if FALSE, a one series data frame is returned. An important difference between tibbles and ordinary data frames is that the default value of drop is FALSE for the former and TRUE for the later. Therefore, with tibbles, the use of [ will always by default return a data frame.

A specific dfidx method is provided for one reason: the column that contains the indexes should be "sticky" (we borrow this idea from the sf package<sup>1</sup>), which means that it should be always returned while using the extractor operator, even if it is not explicitly selected.

<sup>&</sup>lt;sup>1</sup>Pebesma and Bivand (2023) and Pebesma (2018).

```
mn[mn\$unemp > 10, ] | > print(n = 3)
first 3 observations out of 46
 publiccap highway
                       water utilities privatecap
                                                      gsp labor
1 65064.20 27568.50 10218.00
                              27277.69 154806.13 159778 4593.3
2 64751.99 27483.00 10436.04
                              26832.94 157095.80 160856 4530.6
3 25108.79 10618.71 3297.40 11192.68
                                         82361.44 64042 2028.0
             idx
 unemp
1
     11 Illi:1982
2
    11 Illi:1983
3
     12 Indi:1982
~~~ indexes ~~~~
     state
                       region year president
13 Illinois East-North Central 1982
                                       Reagan
14 Illinois East-North Central 1983
                                       Reagan
30 Indiana East-North Central 1982
                                       Reagan
indexes: 1, 1, 2, 2
  mn[mn$unemp > 10, c("highway", "utilities")] |> print(n = 3)
first 3 observations out of 46
  highway utilities
1 27568.50 27277.69 Illi:1982
2 27483.00 26832.94 Illi:1983
3 10618.71 11192.68 Indi:1982
~~~ indexes ~~~~
      state
                       region year president
13 Illinois East-North Central 1982
                                       Reagan
14 Illinois East-North Central 1983
                                       Reagan
30 Indiana East-North Central 1982
                                       Reagan
indexes: 1, 1, 2, 2
  mn[mn\sup > 10, "highway"] |> print(n = 3)
```

All the previous commands extract the observations where the unemployment rate is greater than 10% and, in the first case all the series, in the second case two of them and in the third case only one series.

#### 4.2 Commands that return a series

A series can be extracted using any of the following commands:

```
mn1 <- mn[, "highway", drop = TRUE]
mn2 <- mn[["highway"]]
mn3 <- mn$highway
c(identical(mn1, mn2), identical(mn1, mn3))
## [1] TRUE TRUE</pre>
```

The result is a xseries which inherits the idx column from the data frame it has been extracted from as an attribute:

Note that, except when dfidx hasn't been used with drop.index = FALSE, a series which defines the indexes is dropped from the data frame (but is one of the column of the idx column of the data frame). It can be therefore retrieved using:

```
head(mn$idx$president)
```

[1] Carter Carter Carter Ford Ford Levels: Carter Ford Nixon Reagan

or

```
idx(mn)$president |> head()
```

[1] Carter Carter Carter Ford Ford Levels: Carter Ford Nixon Reagan

or more simply by applying the \$ operator as if the series were a stand-alone series in the data frame :

```
mn$president |> print(n = 3)
```

```
[1] Carter Carter Carter
Levels: Carter Ford Nixon Reagan
--- indexes ----
state region year president
1 Illinois East-North Central 1977 Carter
2 Illinois East-North Central 1978 Carter
3 Illinois East-North Central 1979 Carter
indexes: 1, 1, 2, 2
```

In this last case, the resulting series is a **xseries**, *ie* it inherits the index data frame as an attribute.

#### 4.3 User defined class for extracted series

While creating the dfidx, a pkg argument can be indicated, so that the resulting dfidx object and its series are respectively of class c("dfidx\_pkg", "dfidx") and c("xseries\_pkg", "xseries") which enables the definition of special methods for dfidx and xseries objects. For example, consider the hypothetical pnl package for panel data:

For example, we want to define a lag method for xseries\_pnl objects. While lagging there should be a NA not only on the first position of the resulting vector like for time-series, but each time we encounter a new individual. A minimal lag method could therefore be written as:

```
lag.xseries_pnl <- function(x, ...){</pre>
       .idx < -idx(x)
       class <- class(x)</pre>
       x <- unclass(x)
       id \leftarrow .idx[[1]]
       lgt <- length(id)</pre>
       lagid <- c("", id[- lgt])
       sameid <- lagid == id
       x \leftarrow c(NA, x[-lgt])
       x[! sameid] <- NA
       structure(x, class = class, idx = .idx)
  }
  lmn1 <- stats::lag(mn1)</pre>
  lmn1 \mid > print(n = 3)
[1]
        NA 168627 173767
~~~ indexes ~~~~
     state
                         region year president
1 Illinois East-North Central 1977
                                          Carter
2 Illinois East-North Central 1978
                                          Carter
3 Illinois East-North Central 1979
                                          Carter
indexes: 1, 1, 2, 2
```

```
class(lmn1)
[1] "xseries_pnl" "xseries"
                                  "integer"
  rbind(mn1, lmn1)[, 1:20]
       [,1]
               [,2]
                      [,3]
                              [,4]
                                     [,5]
                                             [,6]
                                                    [,7]
                                                            [,8]
                                                                   [,9]
     168627 173767 173817 165722 157366 163112 145792 148503 154413
mn1
lmn1
         NA 168627 173767 173817 165722 157366 163112 145792 148503
      [,10]
              [,11]
                     [,12]
                             [,13]
                                    [,14]
                                            [,15]
                                                   [,16]
                                                           [,17] [,18]
     163125 161725 166029 159778 160856 173602 178493 183849 68832
lmn1 154413 163125 161725 166029 159778 160856 173602 178493
                                                                    NA
     [,19] [,20]
    71717 72047
mn1
```

Note the use of stats::lag instead of lag which ensures that the stats::lag function is used, even if the dplyr (or tidyverse) package is attached.

## 5 Model building

lmn1 68832 71717

The two main steps in **R** in order to estimate a model are to use the model.frame function to construct a data frame, using a formula and a data frame and then to extract from it the matrix of covariates using the model.matrix function.

### 5.1 Model frame

The default method of model.frame has as first two arguments formula and data. It returns a data frame with a terms attribute. Some other methods exist in the stats package, for example for lm and glm object with a first and main argument called formula. This is quite unusual and misleading as for most of the generic functions in  $\mathbf{R}$ , the first argument is called either  $\mathbf{x}$  or object.

Another noticeable method for model.frame is provided by the Formula package and, in this case, the first argument is a Formula object, which is an extended formula which can contain several parts on the left and/or on the right hand side of the formula.

We provide a model.frame method for dfidx objects, mainly because the idx column should be returned in the resulting data frame. This leads to an unusual order of the arguments, the

data frame first and then the formula. The method then first extract (and subset if necessary the idx column), call the formula/Formula method and then add to the resulting data frame the idx column. The resulting data frame is a dfidx object.

```
mf_mn <- mn |> model.frame(gsp ~ utilities + highway | unemp | labor,
                              subset = unemp > 10)
  mf mn > print(n = 3)
first 3 observations out of 46
     gsp utilities highway unemp labor
                                               idx
1 159778 27277.69 27568.50
                               11 4593.3 Illi:1982
2 160856 26832.94 27483.00
                               11 4530.6 Illi:1983
  64042 11192.68 10618.71
                               12 2028.0 Indi:1982
~~~ indexes ~~~~
                       region year president
     state
1 Illinois East-North Central 1982
                                      Reagan
2 Illinois East-North Central 1983
                                      Reagan
3 Indiana East-North Central 1982
                                      Reagan
indexes: 1, 1, 2, 2
  formula(mf mn)
gsp ~ utilities + highway + unemp + labor + (state + region +
   year + president)
```

Note that the column that contains the indexes is at the end and not at the begining of the returned data frame. This is because the stats::model.response function, which is used to extract the response of a model and is not generic consider that the first column of the model frame is the response.

#### 5.2 Model matrix

model.matrix is a generic function and for the default method, the first two arguments are a terms object and a data frame. In lm, the terms attribute is extracted from the model.frame internally constructed using the model.frame function. This means that, at least in this context, model.matrix doesn't need a formula/term argument and a data.frame, but only a data frame returned by the model frame method, i.e., a data frame with a terms attribute.

We use this idea for the model.matrix method for dfidx object; the only required argument is a dfidx returned by the model.frame function. The formula is then extracted from the dfidx and the Formula or default method is then called. The result is a matrix of class dfidx\_matrix, with a printing method that allows the use of the n argument:

```
mf_mn |> model.matrix(rhs = 1) |> print(n = 5)
# [46 x 3]
  (Intercept) utilities highway
1
               27277.69 27568.50
2
              26832.94 27483.00
3
              11192.68 10618.71
              11141.74 10558.11
4
5
            1 21281.74 19996.38
  mf_mn |> model.matrix(rhs = 2:3) |> print(n = 5)
# [46 x 3]
  (Intercept) unemp labor
1
            1
                 11 4593.3
2
            1
                 11 4530.6
3
                 12 2028.0
4
                 11 2029.5
            1
5
            1
                 12 3442.8
```

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